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NASA Contractor Report 3374

Guide Star Probabilities

Raymond M. Soneira and John N. Bahcall

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INTRODUCTION

We have calculated the probabilities for finding a suitable pair of Guide Stars (GS) in the Fine Guidance System (FGS) of the Space Telescope using the Weistrop north galactic pole star catalog (Weistrop 1972) and classical data on bright star counts (see especially Seares et al. 1925 and Bahcall and Soneira 1980). Many of the considerations and techniques discussed here are relevant for other space astronomy missions. The data are discussed fully in Appendix A of this report (which is a section of a paper by Bahcall and Soneira 1981 on the distribution of bright stars). The available observational data are rather limited but are in good agreement with each other. The definition of our task in this report is contained in Appendix B.

The most important conclusion from this report is that the acquisition probability for guide stars can be greatly increased if the FGS will allow the second member of a guide star pair to be appreciably fainter than the primary (bright) guide star.

a) GS Selection Requirements

The requirements for GS selection that we have used are:

1. Each FGS detector sees an area of sky $\Omega = 69.2$ square arcmin. The geometry for the 3 detectors is shown in figure 1 (which is taken from Groth 1978).

2. The sensitivity of the GS detector may be modeled as a constant between 4660 Å to 7000 Å, and zero outside this range. (This color sensitivity of the GS detector is simil

FINE GUIDANCE SYSTEM
FOCAL PLANE ARRANGEMENT

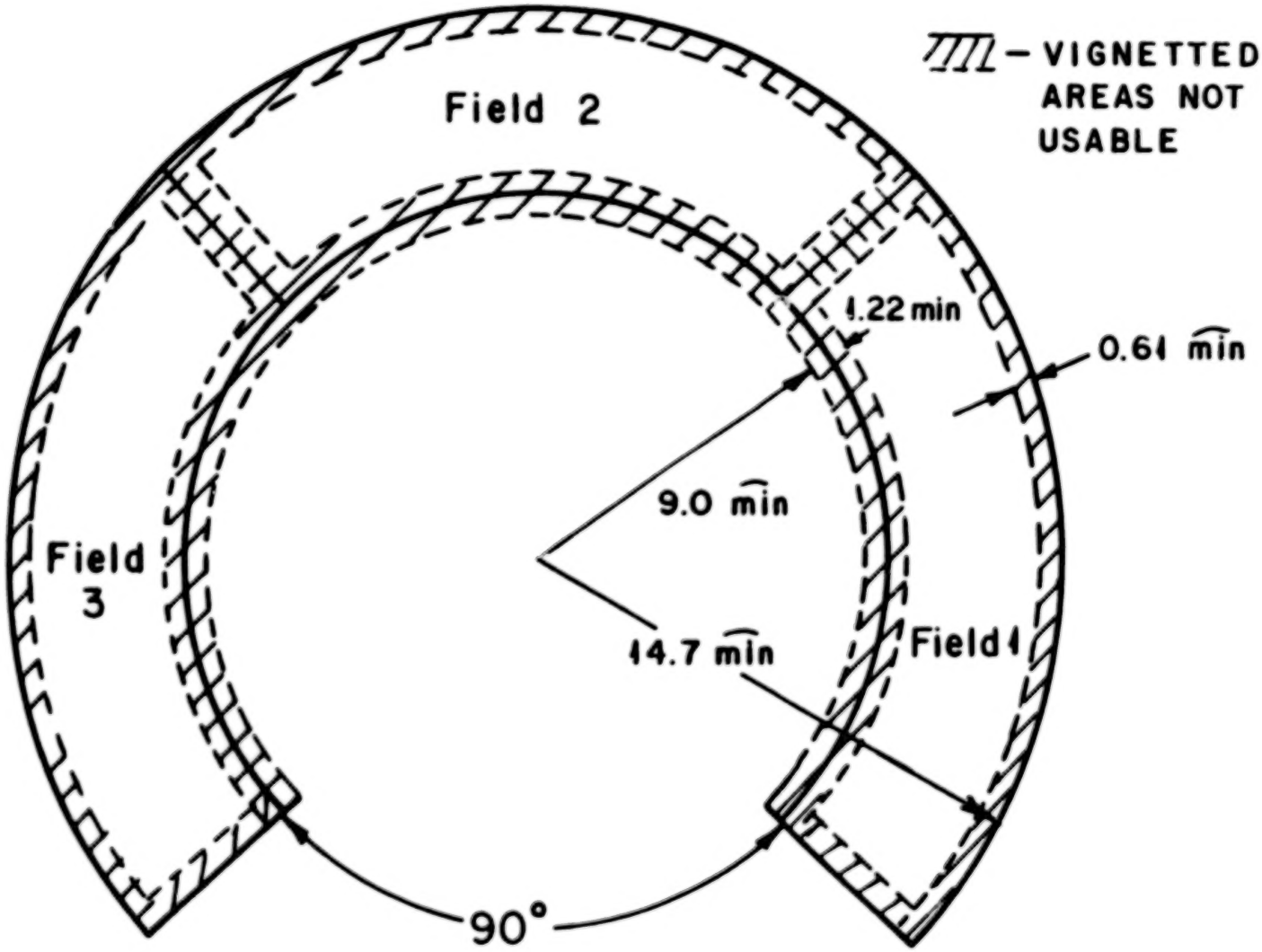

Fig. 1 2

to the Visual band, but has a factor of 2.7 greater energy throughput and 2.8 greater photon throughput for the color distribution of stars brighter than V = 14.0; see Bahcall and Soneira 1981, § III. For V = 15, the energy and photon throughputs are 2.7 and 2.9, respectively.)

3. The magnitude range from the primary and secondary guide stars (i = 1,2) is $9 \leq m_v < m_i$, where $14 \leq m_1 \leq 15$ and $(m_2 - m_1)$ could in some designs be as large as 2 magnitudes.

4. Stars closer than $\theta_e = 1.0$ arcmin to a guide star must have a magnitude separation greater than $\Delta m = 0.5$ mag.

5. A pair of guide stars must be separated by at least $\theta_s = 13.5$ arcmin.

b) Analytic Calculation

An analytic statistical treatment of the selection criteria is useful for gaining an understanding of the GS probabilities. (Calculations using Monte Carlo methods are more accurate and, in some ways, more convenient.)

The probability $P = P(\ell,b,m)$ of finding at least one suitable GS pair in the direction (ℓ,b) with apparent magnitudes brighter than $m = m_1 = m_2$ is given to high accuracy by:

$$P(\ell,b,m) = P_1^3 + (2Q + 1)P_1^2(1 - P_1) \quad , \tag{1}$$

with:

$$P_1 = \frac{1}{\beta}\left[1 - e^{-\beta \mathcal{N} \Omega}\right] \tag{2}$$

$$Q = 1 - \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} P_i P_j (1 - Q_1)^{ij} \quad , \tag{3}$$

$$P_i = \frac{\left(\mathcal{N}'\Omega\right)^i}{i!} e^{-\mathcal{N}'\Omega} \quad , \tag{4}$$

$$\beta = 1 + (10^{\alpha\Delta m} - 1)\frac{\Omega_e}{\Omega} \quad , \text{ assuming} \ , \tag{5}$$

$$\mathcal{N} = C10^{\alpha m} \quad , \ \alpha \text{ and } C \text{ constants} \quad , \tag{6}$$

$$\mathcal{N}' = \mathcal{N}\left(1 - \gamma\mathcal{N}\Omega\right) \ , \tag{7}$$

$$\gamma = \frac{1}{2}\left(10^{\alpha\Delta m} - 10^{-\alpha\Delta m}\right)\frac{\Omega_e}{\Omega} \quad . \tag{8}$$

$\mathcal{N} = \mathcal{N}(\ell,b,m)$ is the number of stars per solid angle brighter than magnitude m satisfying requirement 3 above;

$\mathcal{N}'$ is $\mathcal{N}$ after the application of requirement 4;

Ω is the solid angle seen by one FGS detector, (Ω = 69.2 square arcmin, requirement 1);

θ_e is the minimum angular distance of any star to a GS with a magnitude difference less than Δm (θ_e = 1.0 arcmin, requirement 4);

$\Omega_e = \pi\theta_e^2$ is the area enclosed by θ_e (Ω_e = 3.1 square arcmin);

Δm is the minimum magnitude difference allowed for a star closer than θ_e to a GS. (Δm = 0.5 mag, requirement 4);

θ_s is the minimum angular separation for a pair of GSs (θ_s = 13.5 arcmin, requirement 5);

$Q_1 = Q_1(\theta_s)$ is the (ensemble average) probability that a single pair of GSs, each star having a randomly chosen position within its GS field, are separated by an angular distance $>\theta_s$.

Q_1 for adjacent detector fields is given by the following expression:

$$Q_1 = \frac{\int_{\Delta\theta}^{90°-\Delta\theta} d\theta \int_{R_1}^{R_2} dr_2 \int_{R_1}^{R_2} r_1 r_2 f(\theta, r_1, r_2) dr_1}{\int_{\Delta\theta}^{90°-\Delta\theta} d\theta \int_{R_1}^{R_2} dr_2 \int_{R_1}^{R_2} r_1 r_2 dr_1} \quad , \qquad (9)$$

$$f = \frac{180°-\Delta\theta-\phi-\theta}{90°-2\Delta\theta} \quad , \qquad 0 \leq f \leq 1 \quad , \qquad (10)$$

$$\phi = \cos^{-1}\left[\frac{r_1^2+r_2^2-\theta_s^2}{2r_1r_2}\right] \quad , \qquad (11)$$

where: $R_1 = 10.22$, $R_2 = 14.09$, $\Delta\theta = 2.873°$. R_1, R_2 and $\Delta\theta$ define the non-vignetted detector area. A numerical integration gives $Q_1 = 0.735$. For non-adjacent GS fields, $Q_1 = 1.0$;

α is the logarithmic (base 10) slope of the star counts near m. ($\alpha \approx 0.32$ from the Weistrop data);

β is the correction factor expressing the probability that a GS is rejected due to proximity of another star less than θ_e and Δm away [$\beta = 1.020$];

P_1 is the probability of finding at least one GS in one FGS field satisfying requirement 4;

P_i is the Poisson probability distribution;

The above formulae are approximate for the following reasons:

1. They assume that stars are randomly distributed and therefore ignore star clustering. (Our results suggest that this assumption introduces a ~ 1% overestimate in P);
2. $\mathcal{N}$ is assumed to be exactly an exponential. (This is also an excellent approximation introducing a negligble error);
3. Correction factors due to finite boundaries are ignored in the calculation of β and γ. (This is also an excellent approximation.)

Formulae (1) - (11) were used to calculate the probability of acquiring a GS pair in the memorandum in Appendix B, which describes our definition of the task.

From the Weistrop (1972, 1980) data, the surface density of stars with $9.0 \leq m_v < 14.0$ is 71.5 stars per square degree. The mean number of stars per detector is 1.37. Then $P_1 = 0.74$ and $Q \simeq 0.954$ so that $P = 0.41 + 0.41 = 0.82$.

Note that although the single pair form factor, $Q_1 \simeq 0.75$, the actual form factor $Q \simeq 0.95$ because there are many possible combinations of GS pairs when there is more than one GS per field.

The $\theta_e / \Delta m$ constraint has only a minor effect on P. For $m_v = 14.0$ mag,

$$P \sim 0.84 \left[1 - 0.020 \left(\frac{\Delta m}{0.5}\right) \theta_e^2\right] \quad . \tag{12}$$

For $m_v > 14$, the influence of θ_e and Δm on P decreases monotonically with increasing m_v.

The above value for P is calculated for the Weistrop field centered at ℓ = 65.5°, b = 85.5° (SA 57). At m_v = 14 mag, the star density is expected to be at a minimum slightly away from the Galactic pole in the direction of the galactic anti-center ℓ = 180°, b = 80°. (This is based on the galactic model of Bahcall and Soneira 1980; there is no direct data presently available to confirm this point.) The minimum is not expected to be at the pole because of the galactic spheroidal bulge contribution to the star counts. For ℓ = 180°, b < 80° the increasing star density from the disk overcomes the decrease from the spheroid. Figure 2 is an isodensity contour plot of the integrated star counts to m_v = 14 mag for b $\geq$ 20° calculated from the Bahcall and Soneira model (assuming a Sandage obscuration model, see paper I). (For m_v > 14 the counts increase like $10^{0.32m}$, with only a slight dependence of the slope on latitude and longitude.) The model predicts that the integrated star density to m_v = 14 in the direction ℓ = 180°, b = 80° is $3\frac{1}{2}$% smaller than in the direction ℓ = 65.5°, b = 85.5°. (For m_v = 15, the value is $4\frac{1}{2}$%). This reduces the mean number of stars per detector to 1.32 and P = 0.80. Near m_v = 14, P $\approx$ 0.5(1+0.45n), where n is the mean number of stars per detector. This approximation is accurate to a few percent for n between 1.0 and 1.8 corresponding to the magnitude interval $13.65 \leq m \leq 14.35$.

c) Monte Carlo Calculation

The GS probabilities have also been calculated using a Monte Carlo simulation. The stars in each FGS field were dis-

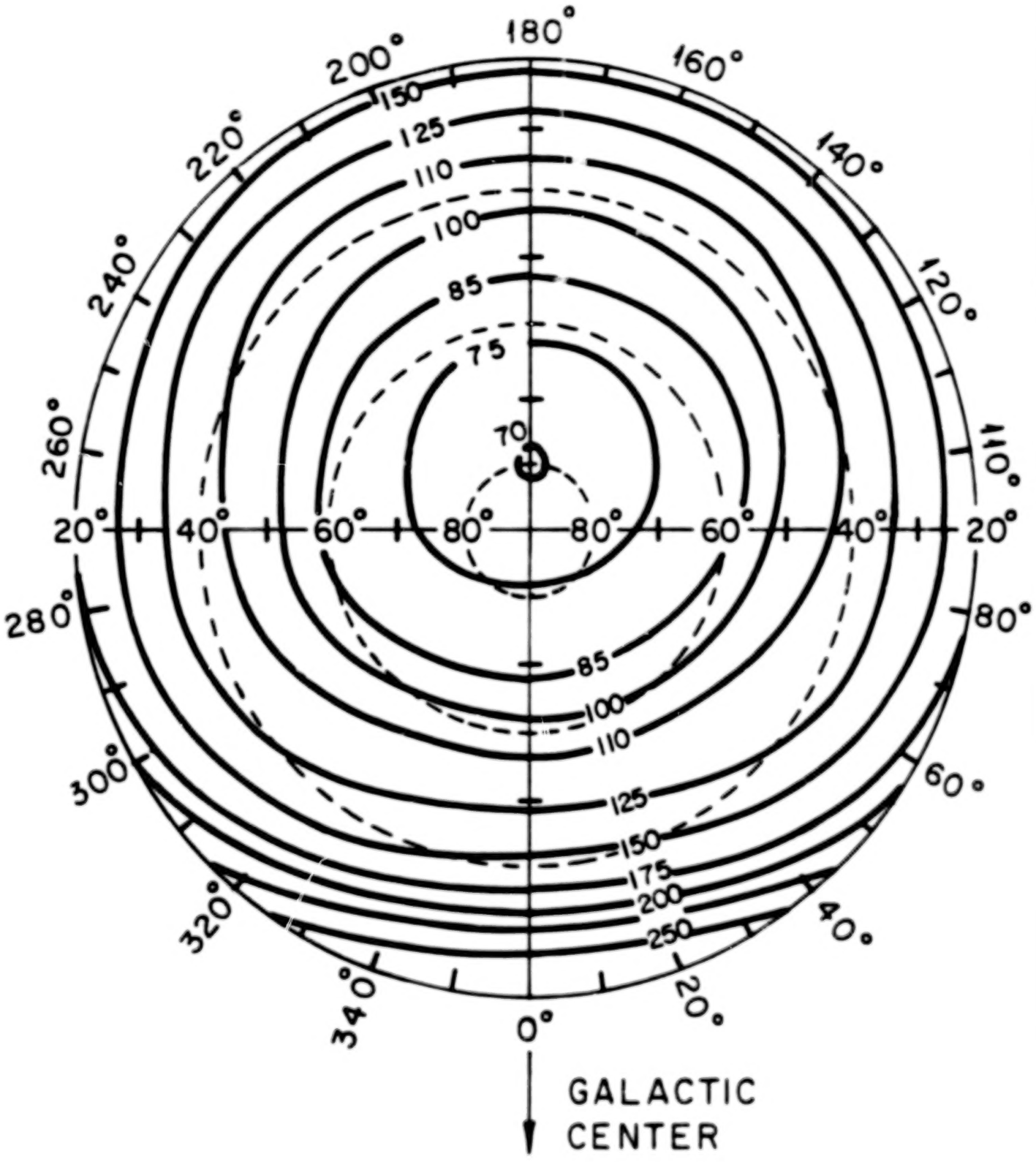
180°
160°
140°
120°
110°
80°
60°
40°
20°
0°
340°
320°
300°
280°
260°
240°
220°
200°
150
125
110
100
85
75
70
85
100
110
125
150
175
200
250
GALACTIC
CENTER

Fig. 2

tributed randomly; the number of stars in each field was chosen from a Poisson distribution with a specified mean number of stars per field and magnitudes were assigned randomly according to a distribution function that results in a number-magnitude relation that increases as $10^{\alpha m}$. The stars were then processed through an algorithm that implements the GS selection requirements (section A, above). This procedure simulates an ST pointing. In order to accumulate good statistics on the distribution of GS pairs, the procedure was repeated several thousand times. The results of this method agree with the analytic calculation to within the statistical uncertainties. Because the results are almost identical to the analysis on the real Weistrop data, we defer discussion to the next section.

d) GS Selection with the Weistrop Data

The Weistrop survey area is ~ 700 times larger than the area of one FGS field and since there are 984 stars brighter than $m_V = 14$ in the catalog, there is a sufficient data base for accurately determining GS statistics empirically near the pole (assuming, of course, the Weistrop field is representative of the star density at its galactic latitude and longitude).

The only significant difference between the results for the Monte Carlo calculation (§ c) and the Weistrop data is the GS rejection due to the proximity of another star: for the random case at $V = 14^m$ it is 2% and, for the Weistrop data it is 3%. The larger rejection factor in the Weistrop data is due to star clustering. The Weistrop catalog presum-

ably identifies all stars that are separated by more than the seeing disk, $\leq$ 2 arcsec (Weistrop 1980); hence only very compact binaries (which may be a problem for the FGS) are missed. (The anticorrelation length for the measured stellar positions is ~ 6 arcsec. The absolute positions of the stars are accurate to ~ 1 arcmin, Weistrop 1980.)

We present in Table 1, GS probabilities for the following combinations of (m_1,m_2): (14,14), (14,14.5), (14,15), (14.5,14.5), (14.5, 15.0), (15.0, 15.0), and (13,15). For each of these pairs of magnitudes we list $n_i = \mathcal{N}(m_i)\Omega$, the mean number of stars per detector, and several GS probability distributions: $P_1(n)$ and $P_2(n)$ are the probabilities for finding n primary or secondary stars per FGS detector before application of any selection constraints; $P_s(n)$ is the probability for finding n suitable secondary guide stars for a primary guide star; and $P_p(n)$ is the probability for finding n suitable guide star pairs per ST pointing.

For the "standard" case of $m_1 = m_2 = 14$, $P_p(\geq 1)$; i.e., the probability of finding at least one suitable GS pair is 0.80, and $P_p(>2) = 0.55$. (This latter value is important for considering backup GS pairs if close binaries are a problem.) For (14,14.5), $P_p(\geq 1) = 0.91$ and $P_p(\geq 10) = 0.35$. Hence allowing the secondary GSs to be 1/2 mag dimmer than the primaries brings about a dramatic increase in the probability of acquiring a suitable GS pair. For (14,15), $P_p(\geq 1) = 0.97$ and $P_p(\geq 1) = 0.87$ for (13,15). Note that for $\mathcal{N}_2 >> \mathcal{N}_1$ P_s follows closely the Poisson distribution for a mean density of $\frac{2}{3}(2Q_1+1)n_2$ and $P_p(\geq 1) \sim 1 - e^{-3n_1}$

For $\mathcal{N}_2 >> \mathcal{N}_1$, the number distribution of GS pairs, P_p, can be significantly reduced if any of the primary Guide Stars is a close binary system. The probability of finding n candidate primary GSs in the three FGS fields taken together is then the statistic of most practical interest because we are virtually guaranteed of finding a secondary GS for each candidate primary GS since $\mathcal{N}_2 >> \mathcal{N}_1$. This statistic (a Poisson distribution for a mean density of $3n_1$) is presented in Table 2 for each of the limiting magnitudes considered in this paper.

As in section b), the "worst case" probabilities need to be corrected from the direction ℓ = 65.5°, b = 85.5° to the density minimum at ℓ = 180°, b = 80°. For the "standard" case of $m_1 = m_2 = 14$, $P_p(\geq 1)$ is then 0.79.

Approximate estimates for the GS probabilities for directions other than the galactic pole can be obtained from figure 2 by matching the surface densities $\mathcal{N}$ for a given direction (ℓ,b) as closely as possible to the entries in Table 1. For $\mathcal{N} \geq 100$ stars per deg^2, coresponding to $m_v \approx 14.5^{mag}$ at the galactic pole, the probability of acquiring a suitable pair of Guide Stars is $\geq$ 95%, sufficiently high to be of little concern. Appropriate Guide Stars will nearly always be available ($P_p(\geq 1) \geq 0.95$) for all galactic latitudes and longitudes below b ~ 40° if $m_1 = m_2 \geq 14^m$ (see Figure 2 and Table 1).

e) Conclusion

The probability for acquiring a suitable pair of Guide Stars in the direction of minimum star density (ℓ = 180°,

$b = 80°$) is estimated in this report to be 0.79 when both Guide Stars are required to be brighter than $m_v = 14^m$, somewhat less than the design specification of 0.85. (The acquisition probability will be further reduced if there are a significant number of close binaries with angular separation $\leq$ 2 arcsec.) In order to meet the design specification with the same brightness limit for both the primary and secondary Guide Stars, one would require $m_1 = m_2 = 14.25$ Visual magnitudes.

If, however, the secondary Guide Star is allowed to be dimmer than the primary, then there can be a significant improvement in the acquisition probability. For $m_1 = 14.0^m$ and $m_2 = 14.5^m$, the Guide Star acquisition probability is 0.90. If we use the maximum allowed brightness difference between the primary and secondary Guide Stars, 2 magnitudes, then the primary star need only be brighter than 13^{th} magnitude (Visual) in order to meet the design specification in the direction of the density minimum near the galactic pole.

REFERENCES

Bahcall, J. N., and Soneira, R. M. 1980, Ap.J. Suppl., in press (Paper I).

_____. 1981, submitted to Ap.J.

Groth, E. J. 1978, "Probabilities of Finding Suitable Guide Stars," in "Use of the WFC as a 'Finder Camera' for the Small Aperture SI's," NASA report.

Seares, F. H., van Rhijn, P. J., Joyner, M. C., and Richmond, M. L. 1925, Ap.J., 62, 320.

Weistrop, D. 1972, A.J., 77, 366.

_____. 1980, private communication.

TABLE 1. Guide Star Probability Distributions *

$m_1 = 14.0$	$m_2 = 14.0$	$m_1 = 14.0$	$m_2 = 14.5$	$m_1 = 14.0$	$m_2 = 15.0$
$n_1 = 1.37$	$n_2 = 1.37$	$n_1 = 1.37$	$n_2 = 1.89$	$n_1 = 1.37$	$n_2 = 2.56$
$\mathcal{N}_1 = 71.5$	$\mathcal{N}_2 = 71.5$	$\mathcal{N}_1 = 71.5$	$\mathcal{N}_2 = 98.3$	$\mathcal{N}_1 = 71.5$	$\mathcal{N}_2 = 133$

n	P_1	P_2	P_s	P_p	n	P_1	P_2	P_s	P_p	n	P_1	P_2	P_s	P_p
0	0.254	0.254	0.113	0.195	0	0.254	0.151	0.052	0.086	0	0.254	0.077	0.018	0.032
1	0.348	0.348	0.236	0.115	1	0.348	0.286	0.137	0.057	1	0.348	0.198	0.067	0.026
2	0.238	0.238	0.262	0.139	2	0.238	0.270	0.212	0.081	2	0.238	0.253	0.134	0.047
3	0.109	0.109	0.196	0.090	3	0.109	0.170	0.211	0.073	3	0.109	0.216	0.181	0.046
4	0.037	0.037	0.111	0.092	4	0.037	0.080	0.173	0.079	4	0.037	0.138	0.181	0.054
5	0.010	0.010	0.052	0.057	5	0.010	0.030	0.110	0.060	5	0.010	0.071	0.159	0.046
6	0.002	0.002	0.020	0.062	6	0.002	0.010	0.058	0.065	6	0.002	0.030	0.111	0.044
7			0.007	0.043	7		0.003	0.029	0.047	7		0.011	0.073	0.048
8			0.002	0.039	8		0.001	0.011	0.057	8		0.004	0.040	0.053
9			0.001	0.029	9			0.004	0.050	9		0.001	0.021	0.047
10				0.027	10			0.001	0.041	10			0.009	0.044
11				0.018	11			0.001	0.035	11			0.004	0.035
12				0.018	12				0.040	12			0.002	0.040
13				0.013	13				0.032	13			0.001	0.039
14				0.012	14				0.026	14				0.034
15				0.011	15				0.018	15				0.029
16				0.007	16				0.020	16				0.035
17				0.006	17				0.015	17				0.027
18				0.006	18				0.015	18				0.028
≥19				0.023	≥19				0.105	≥19				0.247

* See page 10 for definitions.

TABLE 1. Guide Star Probability Distributions* (Continued)

$m_1 = 14.5$	$m_2 = 14.5$	$m_1 = 14.5$	$m_2 = 15.0$	$m_1 = 15.0$	$m_2 = 15.0$
$n_1 = 1.89$	$n_2 = 1.89$	$n_1 = 1.89$	$n_2 = 2.56$	$n_1 = 2.56$	$n_2 = 2.56$
$\mathcal{N}_1 = 98.3$	$\mathcal{N}_2 = 98.3$	$\mathcal{N}_1 = 98.3$	$\mathcal{N}_2 = 133$	$\mathcal{N}_1 = 133$	$\mathcal{N}_2 = 133$

n	P_1	P_2	P_s	P_p	n	P_1	P_2	P_s	P_p	n	P_1	P_2	P_s	P_p
0	0.151	0.151	0.051	0.080	0	0.151	0.077	0.019	0.026	0	0.077	0.077	0.017	0.023
1	0.286	0.286	0.137	0.052	1	0.286	0.198	0.077	0.018	1	0.198	0.198	0.067	0.014
2	0.270	0.270	0.207	0.073	2	0.270	0.253	0.144	0.041	2	0.253	0.253	0.132	0.033
3	0.170	0.170	0.210	0.071	3	0.170	0.216	0.183	0.039	3	0.216	0.216	0.177	0.031
4	0.080	0.080	0.174	0.073	4	0.080	0.138	0.175	0.052	4	0.138	0.138	0.183	0.041
5	0.030	0.030	0.113	0.052	5	0.030	0.071	0.149	0.036	5	0.071	0.071	0.161	0.033
6	0.010	0.010	0.061	0.064	6	0.010	0.030	0.108	0.054	6	0.030	0.030	0.114	0.042
7	0.003	0.003	0.030	0.046	7	0.003	0.011	0.070	0.040	7	0.011	0.011	0.075	0.038
8	0.001	0.001	0.012	0.058	8	0.001	0.004	0.040	0.052	8	0.004	0.004	0.041	0.045
9			0.004	0.046	9		0.001	0.020	0.046	9	0.001	0.001	0.019	0.033
10			0.002	0.046	10			0.009	0.048	10			0.009	0.042
11			0.001	0.032	11			0.004	0.041	11			0.004	0.033
12				0.042	12			0.002	0.042	12			0.002	0.041
13				0.032	13				0.032	13			0.001	0.030
14				0.029	14				0.035	14				0.038
15				0.025	15				0.035	15				0.034
16				0.023	16				0.026	16				0.029
17				0.016	17				0.029	17				0.034
18				0.017	18				0.025	18				0.027
≥19				0.125	≥19				0.286	≥19				0.360

* See page 10 for definitions.

TABLE 1. Guide Star Probability Distributions* (Concluded)

$m_1 = 13$ $\quad m_2 = 15$

$n_1 = 0.684$ $\quad n_2 = 2.56$

$\mathcal{N}_1 = 35.6$ $\quad \mathcal{N}_2 = 133$

n	P_1	P_2	P_s	P_p
0	0.505	0.077	0.019	0.131
1	0.345	0.198	0.065	0.046
2	0.118	0.253	0.140	0.076
3	0.027	0.216	0.187	0.078
4	0.005	0.138	0.181	0.086
5	0.001	0.071	0.154	0.059
6		0.030	0.110	0.062
7		0.011	0.071	0.055
8		0.004	0.039	0.051
9		0.001	0.020	0.050
10			0.010	0.039
11			0.003	0.036
12			0.001	0.036
13			0.001	0.025
14				0.024
15				0.021
16				0.024
17				0.015
18				0.013
≥ 19				0.077

* See page 10 for definitions.

TABLE 2. Number Distribution of Primary Guide Stars For All Three Dectectors.

n	m_o = 13.0 n_1 = 0.684	m_o = 13.5 n_1 = 0.980	m_o = 14.0 n_1 = 1.37	m_o = 14.5 n_1 = 1.89	m_o = 15.0 n_1 = 2.56
0	0.128	0.053	0.016	0.003	0.000
1	0.264	0.155	0.067	0.020	0.004
2	0.270	0.228	0.139	0.055	0.014
3	0.185	0.224	0.190	0.105	0.035
4	0.095	0.165	0.195	0.148	0.067
5	0.039	0.097	0.160	0.168	0.103
6	0.013	0.047	0.110	0.159	0.132
7	0.004	0.020	0.064	0.129	0.144
8	0.001	0.007	0.033	0.091	0.139
9		0.002	0.015	0.058	0.118
10		0.001	0.006	0.033	0.091
11			0.002	0.017	0.063
12			0.001	0.008	0.041
13				0.003	0.024
14				0.001	0.013
15				0.001	0.007
16					0.003
17					0.001
18					0.001
$\geq$19					

APPENDIX A

II. BRIGHT-STAR DENSITY

A. The Best Available Data

The principal source of data that we used for the bright ($\lesssim 16^{mag}$) star densities in paper I was Seares et al. (1925), who list star counts in the direction of the north Galactic pole for magnitudes in the range $4 \leq m_v \leq 18$. The original data from some of the star catalogs used by Seares et al. had large systematic errors (up to 2 magnitudes) that Seares and his collaborators tried to remove. However, a scale error in the "corrected" magnitudes was discovered by Stebbins, Whitford and Johnson (1950). The corrections of Stebbins et al. were applied in paper I, but it was conceivable that some systematic errors remained.

Modern counts of bright star densities have been rare. In paper I, the brightest recent star counts considered were those of Brown (1979). Brown measured integrated star counts in several fields near the north Galactic pole in the magnitude range $17 \leq m_v \leq 20.5$. (The counts of Tyson and Jarvis (1979) extend to 13^{th} magnitude; however, stellar images brighter than 17^{th} magnitude were saturated and hence required an indirect magnitude measurement.)

In this paper we consider additionally star counts from 9^{th} to 16^{th} magnitude obtained from the data of Weistrop (1972, 1980) that was kindly made available to us in the form of

the original computer tape and printout. The Weistrop catalog comes from a field of area 13.5 square degrees, centered on SA57 in the direction of the north Galactic pole. There are 2989 stars in her catalog brighter than V = 16 (244 brighter than V = 12). The catalog extends to V = 20, but magnitude uncertainties appear to increase significantly dimward of 16^{th} magnitude (Faber et al. 1976; Weistrop 1980; King 1980).

According to Weistrop (1980), the magnitude errors for $V \leq 16$ are $\pm 0.12^m$ (one standard deviation). A comparison by Faber et al. 1976 of the Weistrop photographic magnitudes, V_W, to photoelectric magnitudes, V_{PE}, for 23 stars with $V \leq 16$ gives $\langle V_W - V_{PE} \rangle = -0.02 \pm 0.09$. (Systematic errors resulting from minor differences between the photographic and photoelectric pass bands are of concern here only when estimating B-V colors, see below.) King (1980) has compared his own photographic photometry to that of Weistrop. His results for 16 stars with $V \leq 16$ is $\langle V_W - V_K \rangle = +0.01 \pm 0.09$. These estimates all yield a magnitude uncertainty $\sim \pm 0.1^m$. The zero point and magnitude scale appear to be well established for $V \leq 16$.

Figure 1 displays a compilation of all the available differential star densities (number of stars per magnitude per square degree) in the direction of the north Galactic pole (see paper I). The solid curve is the density predicted by our standard Galaxy model of paper I, including a separate contribution from giants (see below). In Table 1 we list the total integrated star counts (stars brighter than the

specified magnitudes limit) for the Weistrop data, the Seares et al. data as reduced to the Visual band in paper I, the calculated densities from our standard Galaxy model (see below), and the densities tabulated in Allen (1973).

B. Allen's Recommended Densities

The star densities listed in Allen (1963, 1973) are typically more than 40% larger than the densities listed above and in paper I for $m \geq 13$. Allen's results have been used as the basis for considering the performance of the Guide Star System for the Space Telescope (see, e.g. , Groth 1978). Because of the importance of the Guide Star System for the entire Space Telescope Observatory, it is necessary to examine the origin of the discrepancies between Allen's numbers and the other results given in Table 1. (Allen 1973 notes that his star densities are 25% larger than the Landolt-Börnstein Tables [Scheffler and Elsässer 1965], his reference number 6, although the same sources were used.) The sources listed by Allen are Seares et al. (1925), Seares and Joyner (1928), and van Rhijn (1929). The Seares and Joyner paper examines star counts for $b \leq 70^{\circ}$, and thus is not relevant for the present discussion. The van Rhijn paper combines star count data of Seares et al. and the Harvard-Durchusterung survey.

A comparison of the Seares et al. and van Rhijn counts in the international photographic band with corresponding counts listed in Allen (1973), shows that Allen must have re-reduced the data introducing small changes, ~10%, (generally) upward in the bright star densities.

Counts in the Visual band are desired for many purposes. In order to transform the photographic band counts into the Visual band, Bahcall and Soneira (1980) used the color equation in Seares et al. Allen used the following (incorrect) conversion from m_{pg} to m_v:

$$\mathcal{N}(\leq m_v) = \mathcal{N}(\leq m_{pg}) \times \left(\frac{\text{total number of stars brighter than } m_v \text{ from } b=0^\circ \text{ to } 90^\circ}{\text{corresponding number for } m_{pg}} \right) \quad (1)$$

This conversion is more incorrect at higher galactic latitudes, since most of the bright stars are counted near the plane of the disk and stellar type (i.e., color) changes systematically with Galactic latitude.

The net discrepancy is unexpectedly large ($\geq$40%) between, on the one hand, either the modern star counts in V (Weistrop 1972, 1980) or the older Seares et al. (1925) data as reduced in paper I, and, on the other hand, the star densities tabulated by Allen (1973). In order to illustrate where in the analysis of Allen the errors appear to have originated, we divide the transformation into several steps. For specificity, we consider the star density in the Visual band at 14th magnitude for which Allen cites 102 stars deg^{-2}. Part of the error appears to result from the different manner in which Allen combined each of the three (tabulated) quantities needed in equation 2 from his two sources. If we apply Allen's transformation (equation 1) to just the Seares et al. data alone, we obtain 94 stars deg^{-2}. (Van Rhijn tabulates only 2 of the 3 quantities, so it is not possible to perform the Allen transformation solely with van Rhijn's data.) If the band transformation is calculated

using the correct color equation described in Seares et al., the density further decreases to 82 stars deg^{-2}. Finally, if the magnitude corrections of Stebbins, Whitford, and Johnson (1950) are applied to the Seares et al. data (as in paper I), we obtain 71 stars deg^{-2} brighter than 14^{th} Visual magnitude in the direction of the north Galactic pole. Weistrop counted 73.5 stars deg^{-2}; the Bahcall-Soneira Galaxy model predicts 70.4 stars deg^{-2} brighter than V = 14. Hence, it appears that Allen's result can be accounted for by 3 systematic errors, each of order 10%, that add "fortuitously" in the same direction.

C. Theoretical Model

In calculating the star counts for the bright apparent magnitudes considered here using the Bahcall-Soneira Galaxy model, it is necessary to separate the contribution of the stars on the giant branch from the more common main sequence stars which completely dominate the counts at faint magnitudes (see § IIa of paper I for a detailed discussion of this point). The giants were not included separately in paper I.

The differential star counts for the giants peak at 11^{th} mag, where they contribute about 20% to the total counts; by 13^{th} mag their contribution has fallen to 3%. We take the fraction of stars, f, on the main sequence in the plane of the disk to be

$$f = \begin{cases} 0.44 \times \exp\left[0.00015(M+8)^{3.5}\right], & M < 3.7 , \\ 1 , & M \geq 3.7 , \end{cases} \qquad (2)$$

which accurately fits the data of Sandage (1957), Schmidt (1959) and McCuskey (1966). A scale height of 250 pc is adopted for the giants (see paper I).

REFERENCES

Allen, C. W. 1963, Astrophysical Quantities, second edition (London: Athlone Press).

_____. 1973, Astrophysical Quantities, third edition (London: Athlone Press).

Bahcall, J. N., and Soneira, R. M. 1980a, Ap.J. Suppl., 44, (Paper I).

Brown, G. S. 1979, A.J., 84, 1647.

Faber, S. M., Burstein, D., Tinsley, B., and King, I. R. 1976, A.J., 81, 45.

Groth, E. J. 1978, "Probabilities of Finding Suitable Guide Stars," in "Use of the WFC as a 'Finder Camera' for the Small Aperture SI's," NASA report.

King, I. R. 1980, private communication.

Kron, R. G. 1978, Ph.D. Thesis, Univ. of California, Berkeley.

McCuskey, S. W. 1966, Vistas in Astronomy, 7, 141.

Peterson, B. A., Ellis, R. S. Kibblewhite, E. J., Bridgeland, M. T., Hooley, T., and Horne, D. 1979, Ap.J. (Letters), 233, L109.

Rhijn, P. J. van 1929, Pub. Kapteyn Astro. Lab. Groningen, No. 43.

Sandage, A. 1957, Ap.J., 125, 422.

Scheffler, H., and Elsässer, H. 1965, Landolt-Börnstein Tables, Group VI, 1, p. 601.

Schmidt, M. 1959, Ap.J., 129, 243.

Seares, F. H. and Joyner, M. C., 1928, Ap.J., 67, 24.

Seares, F. H., van Rhijn, P. J., Joyner, M. C., and Richmond, M. L. 1925, Ap.J., 62, 320.

Stebbins, J., Whitford, A. E., and Johnson, H. L. 1950, Ap.J., 112, 469.

Tyson, J. A. and Jarvis, J. F. 1979, Ap.J. (Letters), 230, L153.

Weistrop, D. 1972, A.J. 77, 366.

_____. 1980, private communciation.

TABLE 1. TOTAL INTEGRATED STAR DENSITIES PER SQUARE DEGREE

V	9	10	11	12	13	14	15	16
Weistrop	1.9	3.7	7.6	18	37	74	135	224
Seares et al.[a]	1.2	3.3	8.0	18	36	71	134	216
Bahcall-Soneira Model	1.5	3.5	7.7	16	34	70	138	254
Allen	1.4	3.6	9.1	21	49	102	186	347

[a] Reduced to the Visual band in paper I.

FIGURE CAPTIONS

FIGURE 1: Differential star densities $\mathcal{A}$ per magnitude per square degree for the Galactic pole. The solid curve is the density predicted by the standard Galaxy model of paper I assuming no obscuration at the pole and including a separate contribution from giants (see § II c). Data from Seares et al. (1925) as reduced to the Visual-band in paper I are plotted as filled-circles, data from Weistrop (1972, 1980) are plotted as filled-triangles, data from Brown (1979) corrected to the pole (see paper I) are plotted as filled-squares, data from Kron (1978) are plotted as open triangles, data from Tyson and Jarvis (1979) are plotted as open circles, and data from Peterson et al. (1979) as open squares. For conversions between magnitude systems, see paper I (and § III).

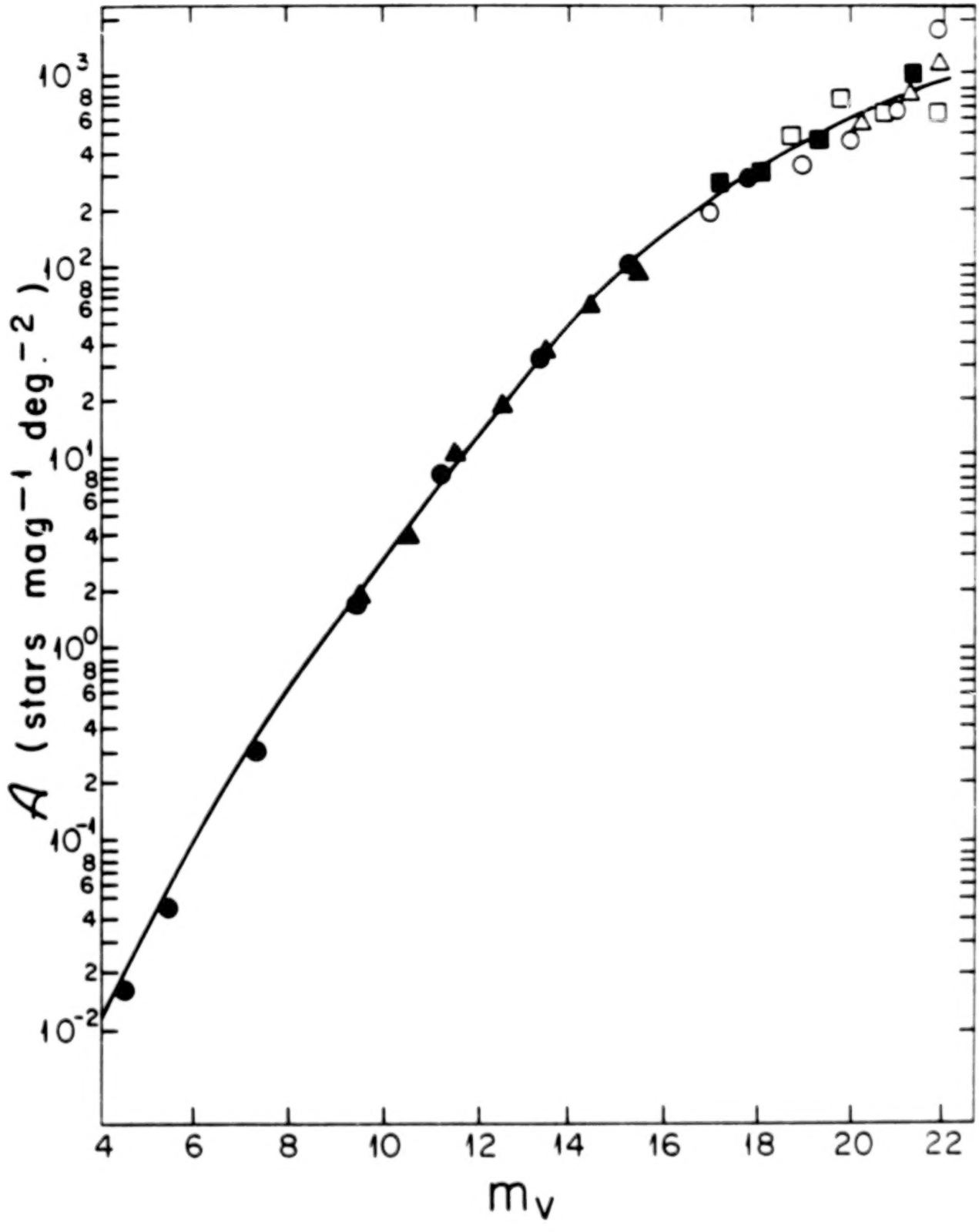

Fig. 1

APPENDIX B

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SCHOOL OF NATURAL SCIENCES

February 13, 1980

MEMORANDUM

To: C. R. O'Dell and W. Jeffreys

From: John N. Bahcall and Raymond Soneira

Subject: Guide-star probabilities.

We are undertaking to supply more accurate estimates for the probabilities of finding guide stars in the FGS. The purpose of this preliminary note is to inform you of the scope of our activity and to present some general numbers. We also wish to indicate our summary of the problem so that you can correct any misunderstandings we may have before we complete a lot of numerical work.

Definition of the Problem

1. Each FGS detector has an area of 69.2 arcmin squared.
2. The magnitude limit is presently specified as $9 \leq m_v \leq X$ where X is in the range 14 to 14.5. (The lower limit, $m_v = 9$, will not be important numerically for our purposes.)
3. The separation of the two required guide stars must exceed 13.5 arcmin.
4. For nearby stars within 1 arcmin of a guide star, the separation in magnitude must be at least 0.5^m. For a guide star of magnitude X, the exclusion applies to magnitude $X \pm 0.5^m$.
5. The sensitivity of the detector may be modeled as constant between 4660 Å to 7000 Å, and zero outside this range. The detector really requires a fixed number of photons per square centimeter per second in this band.

Scope of Our Activity

A. Probabilities. We will calculate probabilities for a convenient grid in latitude and longitude of finding two guide stars in the FGS within the limiting magnitude range of 2 (above) for X = 14.0 to 14.5, in steps of 0.1. For these calculations, we shall use the model star densities in our Paper I[1]. We will calculate the corrections to these proba-

bilities due to constraints (3) and (4) above. We will make similar calculations for m_B.

B. Color Correction. The detector sensitivity is not that of any standard astronomical band. Stars of the same visual magnitude will have different photon fluxes in this band. We will normalize our results to an A0 V star, assuming it has the limiting visual magnitude X of consraint 2 above. Then we will reevaluate the photon fluxes for all stars close to magnitude X using their known intrinsic spectra and our estimate of the relative probabilities of stars of different types from Paper I. This will lead to revised color-corrected estimates of the probabilities of finding guide stars in the FGS.

C. Star Clumping. The bright star N-point correlation function is not known, but it may not be accurate to assume, as in all previous calculations we have seen, that bright stars are Poisson distributed on the sky. We will measure the departures from a Poisson distribution using the star data for 13.5 square degrees near SA57 by Dr. D. Weistrop[2]. We will correct all the above probabilities for the effect of star clumping, if appreciable.

Initial Results

We have completed an initial set of calculations for part A above. We find the following.

a. The minimum star density is slightly displaced from the pole. It actually occurs at $\ell = 180°$, $b = 80°$.

b. At the minimum, we find from Paper I:

$$N(m_v \leq 14.0) = 65 \text{ per square degree.}$$

Subsequent to the writing of Paper I, we have verified our overall normalization (originally based on Seares et al.[3] and recalibrated in 1950 using photoelectric standards by Stebbins et al.[4]) for the star densities by independent comparisons with data of D. Weistrop and I. King[5] for SA57 (latitude 86°). Our results agree with theirs to an accuracy of about 10 percent in the range $m_v = 14$ to 15.

c. The probabilities given in the DRM and by Ed Groth in his recent report are based on Allen's Astrophysical Quantities[6]. Allen uses an ad hoc and incorrect conversion from m_{pg} to m_v, i.e.,

$$N(\leq m_v) = N(\leq m_{pg}) \left(\frac{\text{total number of stars brighter than } m_v \text{ from } b=0° \text{ to } 90°}{\text{corresponding number for } m_{pg}} \right).$$

This conversion is more incorrect the closer one approaches the Galactic poles (where Guide star probabilities are mainly

of interest) since most of the bright stars are counted near the plane of the disk and stellar type (i.e., color) changes systematically with Galactic latitude. Comparable errors, fortuitously all in the same direction, arise from a magnitude error (uncorrected for by Allen) in the Seares et al. counts and by Allen's curious reanalysis of the primary star count data. At the pole, Allen gives

$$N(m_v) = 102 \text{ per degree squared ,}$$

or a factor of 1.6 more than our estimate of 66.5 at the pole.

d. Two other numbers of interest (at the minimum) are:

$$N_v(m_v \leq 14.5) = 93.5 \text{ per square degree and}$$

$$N_v(m_v \leq 15.0) = 132 \text{ per square degree.}$$

e. Criterion 3 above reduces the probability of finding Guide stars by about four percent in adjacent detectors (zero otherwise) and criterion 4 makes a further reduction of about two percent near the poles (3 percent at lower latitudes). The net reduction of the overall probability near the Galactic poles from these criteria is about three percent.

f. The probability of successful Guide star acquisition at the pole is (ignoring color corrections, star clumping and close binaries),

$$0.75 \text{ at } m_v = 14.0$$

$$0.90 \text{ at } m_v = 14.5$$

$$0.95 \text{ at } m_v = 15.0 .$$

g. At $\ell = 0°$, $b = 20°$, $m_v = 14.0$, the corresponding probability is 0.995.

[1] Bahcall, J.N., and Soneira, R.M. 1980, "The Universe at Faint Magnitudes: I. Models for the Galaxy and the Predicted Star Counts," Ap.J. (Supplement), in press. (Paper I).

[2] Weistrop, D. 1972, A.J., 77, 366.
_____. 1980, private communication.

[3] Seares, F.H., van Rhijn, P.J., Joyner, MC., and Richmond, M.L. 1925, Ap.J., 62, 320.

[4] Stebbins, J., Whitford, A.E., and Johnson, H.L. 1950, Ap.J., 112, 469.

[5] King, I. 1980, private communication.

[6] Allen, C.W. 1973, Astrophysical Quantities, The Athlone Press (University of London), 3rd edition. Also see 1st and 2nd editions, 1955 and 1963, respectively.

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Final Report

16. ABSTRACT

Probabilities are calculated for acquiring suitable Guide Stars (GS) with the Fine Guidance System (FGS) of the Space Telescope. A number of the considerations and techniques described here are also relevant for other space astronomy missions. The constraints of the FGS are reviewed. The available data on bright star densities are summarized and a previous error in the literature is corrected. Separate analytic and Monte Carlo calculations of the probabilities are described. A simulation of Space Telescope pointing is carried out using the Weistrop north Galactic pole catalog of bright stars. Sufficient information is presented so that the probabilities of acquisition can be estimated as a function of position in the sky.

The probability of acquiring suitable guide stars is greatly increased if the FGS can allow an appreciable difference between the (bright) primary GS limiting magnitude and the (fainter) secondary GS limiting magnitude.

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